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NUMERICAL CALCULATION OF THE TURBULENT FLOW OF A GAS SUSPENSION IN A TUBE

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A mathematical model for describing the turbulent flow of a gas suspension in a tube has been formulated within the framework of the combined Euler–Lagrange approach. Results of calculations by this model are in good agreement with theoretical data obtained using the continuous model developed by the authors [Teplofizika Aéromekhanika, No. 1, 59–71 (1999)]. *The importance of detailed modeling of the turbulent structure of the carrying medium in the near-wall zone has been shown based on a comparative analysis of computational and experimental data.*

Flows of a gas with suspended particles are widespread in nature and technology. The character of the motion of the disperse phase is, to a great extent, determined by the level of turbulent pulsations of the carrying medium. In turn, the particles entrained by the gas flow facilitate additional dissipation of the turbulence energy of the carrying medium [2, 3]. Therefore, modeling of the mutual effect of the phases at the level of both the average and pulsation (or turbulence) characteristics is of importance. The problem of mathematical description of turbulent near-wall flows of mixtures of a gas and solid particles deserves special attention, since in this case the processes of interaction between the disperse phase and the surfaces bounding the flow become crucial [1, 2] (work on modeling these surfaces is less advanced compared to other branches of the mechanics of aerodisperse media).

At present, in studying dust-laden turbulent flows in channels, wide use is made of the so-called continuous approach [4], where the disperse phase is represented as a continuum with effective properties. The turbulent structure of the carrying medium is, as a rule, calculated using two-parameter differential models. The equations of motion of the disperse phase are closed using transport equations for the second single-point moments of the velocity and concentration pulsations obtained either as a result of implementation of Reynolds averaging [2, 5, 6] or from the kinetic equation for the probability density function of the coordinate and velocity distribution of the particles [7, 8]. The character of the interaction between the particles and the channel walls is allowed for by the empirically determined values of the coefficients of accommodation or restitution [4] of the velocity of the particles after their collision with the surface.

Along with these models, models constructed within the framework of the Euler–Lagrange method of describing the flow of a mixture have been developed [9, 11]. In this method, the flow of the carrying medium is modeled by the Reynolds equations with source terms that allow for interphase interaction, while the motion of the disperse elements is determined within the framework of the single-particle method, which makes it possible to follow in detail the change in the actual dynamic characteristics of the particles in their motion in the flow and as a result of collision with the surface. Here the turbulent structure of the carrying medium is, as a rule, calculated based on the two-parameter k–ε model of turbulence [10, 11]. The effect of the pulsations of the gas velocity on the motion of individual particles is allowed for by the method of stochastic modeling [10].

Thus, in constructing a mathematical model of the turbulent flow of a gas with suspended solid particles in a tube, one should pay attention to the choice of a differential model of turbulence capable of representing adequately the level of turbulent pulsations of the carrying medium. This work is aimed at evaluating,

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within the framework of the Euler–Lagrange method of description of mixture flow, the efficiency of the differential models of turbulent transfer of the class of *k*-theories of turbulence as to representation of the effects of the interaction between the particles and the surface.

A steady-state axisymmetric turbulent flow of a viscous gas with a small amount of spherical solid monodisperse particles is considered. The motion occurs in a vertical tube of constant cross section. The equations of motion of the carrying medium written in the boundary-layer approximation have the following form:

$$\frac{\partial \rho w_x}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho w_r) = 0 \,, \tag{1}$$

$$\rho w_x \frac{\partial w_x}{\partial x} + \rho w_r \frac{\partial w_x}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\,(\mu_{\rm g} + \mu_{\rm T})\,\frac{\partial w_x}{\partial r} \right] - \frac{\partial P}{\partial x} - F_x \pm \rho g \,, \tag{2}$$

$$\frac{\partial P}{\partial r} = 0 \,, \tag{3}$$

where F_x is the force of interphase interaction.

Within the framework of the Lagrangian representation of particle motion, use is made of an approach [9] according to which all particles entering the tube are divided into groups with the same characteristics. The motion of the distinguished groups of particles is analyzed by the position and velocity of their representative, which are determined from the solution of the following system of equations [2, 11]:

$$\frac{d\vec{\tilde z}_i}{dt} = \vec{\tilde w}_i \,, \tag{4}$$

$$m_{\rm p}\frac{d\vec{\tilde w}_i}{dt} = \vec F_{{\rm d}i} \pm m_{\rm p}\vec g + \vec F_{{\rm M}i} + \vec F_{Ii} \,, \tag{5}$$

$$I_{\rm p}\frac{d\vec{\tilde \omega}_i}{dt} = \vec{\tilde M}_i \,. \tag{6}$$

The components of the force acting on a particle, which are represented by the right-hand side of Eq. (5), are determined by the relations

$$\vec F_{{\rm d}i} = \frac{3}{4}\frac{\rho m_{\rm p}}{\rho_{\rm p} D_i} c_{{\rm d}i}\,(\vec{\tilde w} - \vec{\tilde w}_i)\,|\vec{\tilde w} - \vec{\tilde w}_i| \,,$$

$$c_{{\rm d}i} = \begin{cases} \dfrac{24}{{\rm Re}_i}(1 + 0.15\,{\rm Re}_i^{0.687}) \,, & {\rm Re}_i \le 1000 \,, \quad {\rm Re}_i = \rho D_i\,|\vec{\tilde w} - \vec{\tilde w}_i|/\mu_{\rm g} \,, \\ 0.44 \,, & {\rm Re}_i > 1000 \,, \end{cases}$$

$$\vec F_{{\rm M}i} = \frac{\pi}{8} D_i^3\,\rho\,[(\vec{\tilde\omega} - \vec{\tilde\omega}_i)\times(\vec{\tilde w} - \vec{\tilde w}_i)] \,, \quad \vec{\tilde\omega} = \frac{1}{2}\,{\rm rot}\,\vec{\tilde w},$$

$$\vec F_{Ii} = \left(0\,, \frac{\tilde w_{\varphi i}^2}{r}\,, -\frac{\tilde w_{ri}\tilde w_{\varphi i}}{r}\right), \quad \vec{\tilde w}_i = (\tilde w_{xi}, \tilde w_{ri}, \tilde w_{\varphi i}) \,, \quad \vec M_i = \pi\mu_{\rm g} D_i^3\,(\vec{\tilde\omega} - \vec{\tilde\omega}_i) \,. \tag{7}$$

The conditions of the turbulent flow of a gas suspension in a tube studied in this work make it possible to simplify the form of Eqs. (4)-(6). By virtue of the axial symmetry of the averaged flow and the adopted assumptions typical of the boundary layer, in the part of the problem formulation related to the dynamics of the particles one can restrict oneself to the following relations:

$$\frac{d\tilde{x}_i}{dt} = \tilde{w}_{xi}\,, \quad \frac{d\tilde{r}_i}{dt} = \tilde{w}_{ri}\,, \tag{8}$$

$$\frac{d\tilde{w}_{xi}}{dt} = \frac{\tilde{w}_x - \tilde{w}_{xi}}{\tau_i} \pm g + \lambda\,(\tilde{w}_r - \tilde{w}_{ri})\,(\tilde{\omega}_{\varphi i} - \Omega_\varphi)\,, \tag{9a}$$

$$\frac{d\tilde{w}_{ri}}{dt} = \frac{\tilde{w}_r - \tilde{w}_{ri}}{\tau_i} + \frac{\tilde{w}_{\varphi i}^2}{\tilde{r}_i} + \lambda\,[(\tilde{w}_\varphi - \tilde{w}_{\varphi i})\,\tilde{\omega}_{xi} - (\tilde{w}_x - \tilde{w}_{xi})\,(\tilde{\omega}_{\varphi i} - \Omega_\varphi)]\,, \tag{9b}$$

$$\frac{d\tilde{w}_{\varphi i}}{dt} = \frac{\tilde{w}_\varphi - \tilde{w}_{\varphi i}}{\tau_i} - \frac{\tilde{w}_{ri}\tilde{w}_{\varphi i}}{\tilde{r}_i} - \lambda\,(\tilde{w}_r - \tilde{w}_{ri})\,\tilde{\omega}_{xi}\,, \tag{9c}$$

$$\frac{d\tilde{\omega}_{xi}}{dt} = -\frac{1}{\tau_\omega}\,\tilde{\omega}_{xi}\,, \quad \frac{d\tilde{\omega}_{\varphi i}}{dt} = -\frac{1}{\tau_\omega}\,(\tilde{\omega}_{\varphi i} - \Omega_\varphi)\,, \tag{10}$$

$$\tau_i = \rho_p D_i^2/(18\mu_g f_i)\,, \quad f_i = c_{di}\,\mathrm{Re}_i/24\,, \quad \tau_\omega = \rho_p D_i^2/(60\mu_g)\,,$$

$$\lambda = 0.75\rho/\rho_p\,, \quad \Omega_\varphi = -\frac{1}{2}\frac{\partial \tilde{w}_x}{\partial r}\,.$$

It is assumed in (9)-(10) that $\tilde{\omega}_{ri} = 0$, and the differential equation (10) is used for $\tilde{\omega}_{xi}$, since, in collision of a particle with the wall, the value of the component of the angular velocity $\tilde{\omega}_{xi}$, in contrast to $\tilde{\omega}_{ri}$, can change substantially.

Modeling of the actual values of the components of the gas velocity $\vec{\tilde{w}} = \vec{w} + \vec{w}\,'$ within the framework of the Lagrangian representation requires reliable information about the averaged turbulent characteristics of the carrying medium. In this work, we make a comparative analysis of different turbulence models that are used for calculating the kinematic characteristics of the carrying medium in a gas-suspension flow. For this purpose, first of all, we use the one-parameter Vasil'ev–Kvon model of turbulence [12]

$$\rho w_x \frac{\partial k}{\partial x} + \rho w_r \frac{\partial k}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\,(\mu_g + 0.4\mu_T)\frac{\partial k}{\partial r}\right] + \mu_T\left(\frac{\partial w_x}{\partial r}\right)^2 -$$

$$- \frac{3.93\,(\mu_g + 0.4\mu_T)\,k}{L^2} - \rho\varepsilon_{add}\,,$$

$$\frac{L\,(r)}{r_0} = 0.37 - 0.24\left(\frac{r}{r_0}\right)^2 - 0.13\left(\frac{r}{r_0}\right)^4,$$

$$\mu_T = 0.2\rho\,\sqrt{k}\,L\,[1 - \exp\,(-\sigma_1 \mathrm{Re}_T^{*2}) + \sigma_3\sqrt{\mathrm{Re}_T^*}\,\exp\,(-\sigma_2 \mathrm{Re}_T^{*2})]\,,$$

$$\mathrm{Re}_{\mathrm{T}}^{*} = \rho\,\sqrt{k}\,L/\mu_{\mathrm{g}}\,,\quad \sigma_1 = 2.1\cdot 10^{-4}\,,\quad \sigma_2 = 4\cdot 10^{-4}\,,\quad \sigma_3 = 0.02\,, \tag{11}$$

and the modifications of the k–ε model by Launder–Sharma [13], Chien [14], and Lin [15]

$$\rho w_x \frac{\partial k}{\partial x} + \rho w_r \frac{\partial k}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\mu_{\mathrm{g}} + \frac{\mu_{\mathrm{T}}}{\sigma_k}\right)\frac{\partial k}{\partial r}\right] + \mu_{\mathrm{T}}\left(\frac{\partial w_x}{\partial r}\right)^2 -$$

$$- \rho\,(\bar{\varepsilon} + \hat{\varepsilon}) + \Pi_k - \rho\varepsilon_{\mathrm{add}}\,, \tag{12a}$$

$$\rho w_x \frac{\partial \bar{\varepsilon}}{\partial x} + \rho w_r \frac{\partial \bar{\varepsilon}}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\mu_{\mathrm{g}} + \frac{\mu_{\mathrm{T}}}{\sigma_{\varepsilon}}\right)\frac{\partial \bar{\varepsilon}}{\partial r}\right] + c_1 f_1 \mu_{\mathrm{T}}\left(\frac{\partial w_x}{\partial r}\right)^2 \frac{\bar{\varepsilon}}{k} -$$

$$- \rho c_2 f_2\,(\mathrm{Re}_{\mathrm{T}})\,\frac{\bar{\varepsilon}^2}{k} + \Pi_{\varepsilon} + E - \rho\Phi_{\mathrm{add}}\,, \tag{12b}$$

$$\mu_{\mathrm{T}} = c_{\mu} f_{\mu}\,(\mathrm{Re}_{\mathrm{T}})\,\frac{k^2}{\bar{\varepsilon}}\,. \tag{12c}$$

which have gained widespread acceptance in calculations of internal flows. The constants and functions of (12) are given in Table 1.

The values of the pulsation component of the gas velocity $\vec{w}\,'$ are modeled by the random Gauss functions $w_x' = \zeta_x\sqrt{\overline{w_x'^2}}$, $w_r' = \zeta_r\sqrt{\overline{w_r'^2}}$, and $w_{\varphi}' = \zeta_{\varphi}\sqrt{\overline{w_{\varphi}'^2}}$, where $\overline{w_x'^2} = \overline{w_r'^2} = \overline{w_{\varphi}'^2} = 2k/3$. The random numbers ζ_i obey the normal Gauss distribution with mean equal to zero and dispersion equal to 1. Their independent values are determined each time anew after a lapse of the interval of time τ_{T}, which is selected as the least between the lifetime of a turbulent vortex τ_{lag} and the time of residence of a particle in this vortex τ_{rel} [10]:

$$\tau_{\mathrm{lag}} = \frac{L}{\sqrt{2k/3}}\,,\quad \tau_{\mathrm{rel}} = \frac{L}{|\tilde{\vec{w}} - \tilde{\vec{w}}_i|}\,,\quad L = \frac{k^{3/2}}{\varepsilon}\,. \tag{13}$$

The reverse effect of the disperse phase on the carrying medium can be found from the results of calculation of the trajectories of the representatives of all groups of particles [11]:

$$\vec{F} = \frac{1}{N_{\mathrm{T}} V^{(k,j)}} \sum_{i=1}^{N} (\vec{F}_{\mathrm{d}i} + \vec{F}_{\mathrm{M}i})\,\frac{\dot{m}_i}{m_{\mathrm{p}}}\,\Delta t_i\,,$$

$$\rho\varepsilon_{\mathrm{add}} = \frac{1}{N_{\mathrm{T}} V^{(k,j)}} \sum_{i=1}^{N} [(\vec{F}_{\mathrm{d}i} + \vec{F}_{\mathrm{M}i})\,\tilde{\vec{w}}]\,\frac{\dot{m}_i}{m_{\mathrm{p}}}\,\Delta t_i - (\vec{F}, \vec{w})\,,\quad \Phi_{\mathrm{add}} = 1.87\,\varepsilon_{\mathrm{add}}\frac{\varepsilon}{k}\,. \tag{14}$$

It is assumed in the combined Euler–Lagrange description of mixture motion that the region of investigation is divided into a finite number of nonintersecting control volumes $V^{(k,j)}$ [16] and the characteristics of the reverse effect in (14) have definite values for each volume $V^{(k,j)}$.

The boundary conditions for the system of equations that defines the motion of the carrying medium have the form

$$x = 0:\ w_x = U_0\,,\quad w_r = 0\,,\quad k = k_0\,,\quad \bar{\varepsilon} = \bar{\varepsilon}_0\,,$$

TABLE 1. Constants and Functions of the Two-Parameter k–ε Model of Turbulence

Notation	Launder–Sharma model [13]	Chen's model [14]	Lin's model [15]
c_μ	0.09	0.09	0.09
f_μ	$\exp\left(\dfrac{-3.4}{(1+0.02\,\mathrm{Re_T})^2}\right)$	$1-\exp(-0.0115y^+)$	$1-\exp\left(\dfrac{-y_\lambda}{100}-\dfrac{8y_\lambda^3}{1000}\right)$
σ_k	1.0	1.0	$1.4-1.1\exp\left(-\dfrac{y_\lambda}{10}\right)$
σ_ε	1.3	1.3	$1.3-\exp\left(-\dfrac{y_\lambda}{10}\right)$
Π_k	0	0	$-\dfrac{1}{2r}\dfrac{\partial}{\partial r}\left(r\mu_g\dfrac{k}{\varepsilon}\dfrac{\partial\hat{\varepsilon}}{\partial r}\right)$
Π_ε	0	0	$-\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\mu_g\dfrac{\varepsilon}{k}\dfrac{\partial k}{\partial r}\right)$
$\hat{\varepsilon}$	$2\mu_g\left(\dfrac{\partial\sqrt{k}}{\partial r}\right)^2$	$2\mu_g\dfrac{k}{y^2}$	$2\mu_g\left(\dfrac{\partial\sqrt{k}}{\partial r}\right)^2$
E	$2\dfrac{\mu_g\mu_T}{\rho}\left(\dfrac{\partial^2u}{\partial r^2}\right)^2$	$-2\mu_g\left(\dfrac{\bar{\varepsilon}}{y^2}\right)\exp(-0.5y^+)$	0
f_1	1.0	1.0	1.0
f_2	$1-0.3\exp(-\mathrm{Re_T^2})$	$1-0.22\exp\left(-\dfrac{\mathrm{Re_T^2}}{36}\right)$	$1-0.22\exp\left(-\dfrac{\mathrm{Re_T^2}}{36}\right)$
c_1	1.44	1.35	1.44
c_2	1.92	1.8	1.92
$\mathrm{Re_T}$	$\mathrm{Re_T}=(k^2)/(\mu_g\bar{\varepsilon})$	$\mathrm{Re_T}=(k^2)/(\mu_g\bar{\varepsilon})$	$\mathrm{Re_T}=(k^2)/(\mu_g\bar{\varepsilon})$
		$y^+=(yu_\tau)/(\mu_g)$	$y_\lambda=(y\sqrt{\varepsilon})/(\sqrt{\mu_g k})$

$$r=0:\ \frac{\partial w_x}{\partial r}=w_r=\frac{\partial k}{\partial r}=\frac{\partial\bar{\varepsilon}}{\partial r}=0\,,$$
$$r=r_0:\ w_x=w_r=k=\bar{\varepsilon}=0\,. \qquad (15)$$

In modeling the interaction (collision) between the particles and the channel walls, use is made of the approach developed by Sommerfeld [11], which allows for the stochastic character of the roughness distribution on the inner surface of the tube. According to this approach, the angle of collision of a particle with the wall is the sum of the angle between the direction of flight of the particle and a straight line parallel to the generatrix of the channel surface α_x and the random angle of slope of the tangent to the rough wall of the tube $\Delta\gamma_x\cdot\xi_{wx}$ (Fig. 1):

$$\tilde{\alpha}_x=\alpha_x+\Delta\gamma_x\cdot\xi_{wx}\,,\quad \tilde{\alpha}_x>0\,, \qquad (16)$$

where $\Delta\gamma_x$ is the mean value of the angle of roughness of the tube surface in the x direction; ξ_{wx} is a random number obeying the normal Gauss distribution with mean equal to 0 and dispersion equal to 1.

A similar technique was used to allow for the roughness of the tube surface in advancing in the circumferential direction, i.e., along φ, which made it possible to substantially improve the quality of the results obtained.

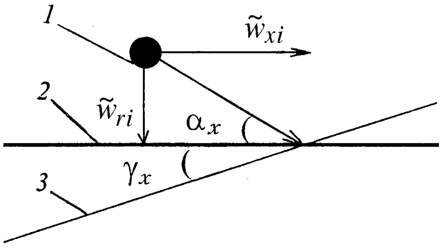

Fig. 1. Sketch of the modeling of the collision of particles with the wall surface: 1) particle; 2) wall; 3) virtual surface.

The change in the components of the linear and angular velocities in collision of the particles with the wall depends on whether the interaction between the particles and the solid surface takes place with or without slipping [17]. Collision without slipping occurs when

$$\tilde{U}_{i1} < \frac{7}{2}\mu_0 (1+e)(\tilde{w}_{ri})_1 , \tag{17}$$

where $\tilde{U}_{i1} = \left(\sqrt{(\tilde{w}_{xi} + D_i\tilde{\omega}_{\varphi i}/2)^2 + (\tilde{w}_{\varphi i} - D_i\tilde{\omega}_{xi}/2)^2}\right)_1$; μ_0 is the static coefficient of friction, which depends on the properties of the material of the particles and the wall; e is the coefficient of restitution of the normal component of the particle velocity upon collision with the wall, which is found experimentally. In such an interaction, the dynamic parameters of the particles obey the following relations:

$$(\tilde{w}_{xi})_2 = \frac{5}{7}\left(\tilde{w}_{xi} + \frac{D_i}{5}\tilde{\omega}_{\varphi i}\right)_1 , \quad (\tilde{w}_{\varphi i})_2 = \frac{5}{7}\left(\tilde{w}_{\varphi i} - \frac{D_i}{5}\tilde{\omega}_{xi}\right)_1 ,$$

$$(\tilde{w}_{ri})_2 = -e(\tilde{w}_{ri})_1 , \quad (\tilde{\omega}_{\varphi i})_2 = \frac{2}{D_i}(\tilde{w}_{xi})_2 , \quad (\tilde{\omega}_{xi})_2 = \frac{-2}{D_i}(\tilde{w}_{\varphi i})_2 . \tag{18}$$

For the case of collision with slipping, according to [11] we take

$$(\tilde{w}_{xi})_2 = (\tilde{w}_{xi})_1 - \mu_d\varepsilon_{xi}(1+e)(\tilde{w}_{ri})_1 , \quad (\tilde{w}_{ri})_2 = -e(\tilde{w}_{ri})_1 ,$$

$$(\tilde{w}_{\varphi i})_2 = (\tilde{w}_{\varphi i})_1 - \mu_d\varepsilon_{\varphi i}(1+e)(\tilde{w}_{ri})_1 , \quad \varepsilon_{xi} = \left(\tilde{w}_{xi} + \frac{D_i}{2}\tilde{\omega}_{\varphi i}\right)_1 \Big/ \tilde{U}_{i1} ,$$

$$(\tilde{\omega}_{\varphi i})_2 = (\tilde{\omega}_{\varphi i})_1 + 5\mu_d(1+e)(\tilde{w}_{ri})_1 \frac{\varepsilon_{xi}}{D_i} , \quad \varepsilon_{\varphi i} = \left(\tilde{w}_{\varphi i} - \frac{D_i}{2}\tilde{\omega}_{xi}\right)_1 \Big/ \tilde{U}_{i1} , \tag{19}$$

$$(\tilde{\omega}_{xi})_2 = (\tilde{\omega}_{xi})_1 - 5\mu_d(1+e)(\tilde{w}_{ri})_1 \frac{\varepsilon_{\varphi i}}{D_i} .$$

Here μ_d is the dynamic coefficient of friction, which depends on the properties of the material of the particles and the wall. In this work, its values are determined with allowance for the data of [18]. The coefficient of accommodation e, which enters (19), is calculated according to [19]:

$$e = 1 - \left(1 - \exp\left(-0.1\,|(\vec{\tilde{w}}_i)_1|^{0.61}\right)\right)\sin(\tilde{\alpha}_x) . \tag{20}$$

The problem formulated was solved numerically by the method of finite differences on nonuniform difference grids that bunched toward the tube inlet and toward the wall so that no less than five nodal points of the grid were in the viscous sublayer. The governing differential equations and the boundary conditions for them were approximated by implicit finite-difference analogs with second order of accuracy along the radial coordinate and first order of accuracy along x. The obtained systems of linear algebraic equations were solved

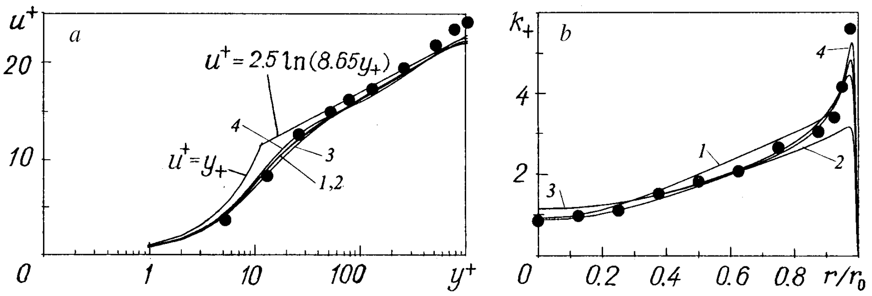

Fig. 2. Comparison of the test data of [22] (points) with predicted values of the velocity of the carrying medium in logarithmic coordinates (a) and with calculated values of the turbulence energy of gas k_+ in stabilized motion of a dust-free gas flow in a tube (b) (Re_D = 32,000; r_0 = 0.02 m); calculation based on the turbulence models of: 1) [12]; 2) [13]; 3) [14]; 4) [15].

TABLE 2. Coefficients of Friction c_f

Models	[12]	[13]	[14]	[15]	Blasius formula
$M = 0$, $Re_D = 32\,000$	0.00586	0.00559	0.00609	0.00565	0.00590

by efficient factorization techniques. The pressure gradient and the longitudinal velocity of the gas were calculated using the algorithm of L. M. Simuni [20]. The system of ordinary differential equations that describes the motion of the particles was solved by the Runge–Kutta method of fourth or fifth order with automatic selection of the step of integration over time [21]. By virtue of the nonlinearity and interrelation of the equations of the mathematical model, use was made of global iterations with lower relaxation with a coefficient of 0.1-0.2 being used in calculation of the terms of (14) that model the reverse effect of the disperse phase on the carrying medium. The overall computational scheme involves the following stages:

1) numerical solution for a single-phase flow that will serve as the initial approximation for further calculations;

2) step-by-step calculation of the trajectories of motion of the representative particles of the groups with simultaneous calculation of the functions of the reverse effect $\vec{F}$, ε_{add}, and Φ_{add};

3) solution of the equations of motion for the carrying medium in the presence of the particles;

4) if the iteration process is not reached, passage to stage 2 occurs.

To obtain a reliable picture of the averaged flow using the considered method of modeling of dust-laden flows, one must carry out numerous calculations of the trajectories of motion of the particles at each iteration. In this work, the number of flights of the representative particles at each step was 3000–8000, depending on the regime of mixture flow. Numerical integration of the equations of motion of the carrying medium was carried out on difference grids containing 50 nodes in the transverse direction and 40 in the longitudinal.

Figure 2 presents distributions of the relative velocity u^+ and the energy of gas turbulence k_+ ($u^+ = w_x/v_*$ and $k_+ = k/v_*^2$, $v_* = \sqrt{\tau_w/\rho}$ is the dynamic velocity) along the tube radius that were obtained numerically based on the considered models of turbulence for the case of a steady-state stabilized single-phase flow in a round tube. As follows from the figure, all tested models of turbulence satisfactorily predict the values of the velocity both in the laminar sublayer and the buffer zone and in the region of the logarithmic velocity profile. As to the turbulence energy, here one must note the following. In spite of the acceptable correlation between the obtained values of $k_+ = k/v_*^2$ and the test data of [22] in the turbulent flow core, a substantial scatter of the calculation results is observed near the wall. The lowest values of k_+ are given by the Launder–Sharma version [13]. The modification of the k–ε model presented in [14] predicts the maximum value of the turbulence energy for moderate Reynolds numbers better, but at the same time it gives overestimated values of k_+ on the tube axis. The best agreement with the data of the measurements made in [22] is observed when the one-parameter model of [12] and the Lin version [15] of the two-parameter k–ε model of turbulence are used. The

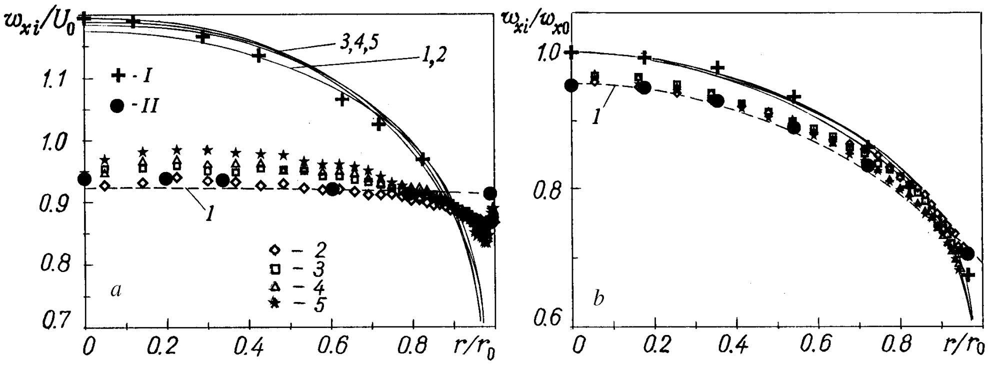

Fig. 3. Calculated and measured [a) [23]; b) [24]] profiles of the velocity of the carrying medium (I) and the dispersed phase (II) in stabilized flow of a gas suspension in a tube: 1) calculation by the continuous model of [1]; 2-5) calculation based on the turbulence models [2) [12]; 3) [15]; 4) [14]; 5) [13]].

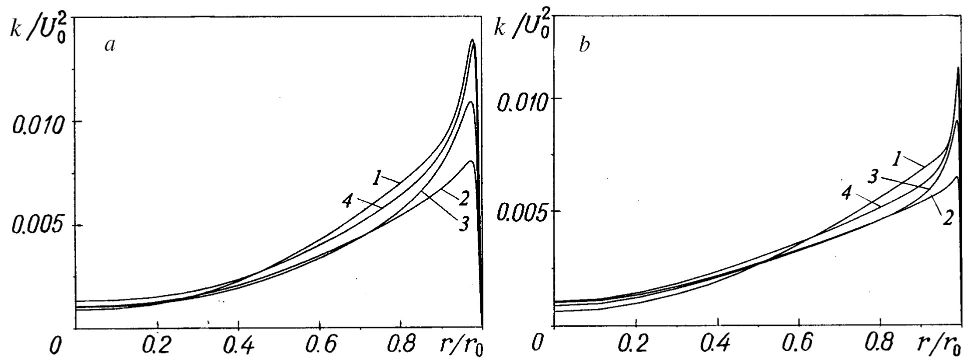

Fig. 4. Comparison of the values of the turbulence energy of gas calculated under the same experimental conditions [a) [23]; b) [24]] in stabilized flow of a gas suspension in a horizontal tube; calculation based on the turbulence models of: 1) [12]; 2) [13]; 3) [14]; 4) [15].

conclusion drawn is confirmed by Table 2, where calculated values of the coefficient of friction $c_f = 2\tau_w/(\rho U_0^2)$ are presented.

Thus, in numerical study of turbulent single-phase flows in tubes, the version of [15] is preferable among the considered modifications of the k–ε model, but the model of [12] with geometric relation (11) for the turbulence scale L can also be used.

For two-phase turbulent flows in a tube the models were tested on the experimental data of [23] ($\text{Re}_\text{D} = \rho U_0 2r_0/\mu_\text{g}$ = 36,250; D_i = 65 μm; r_0 = 0.008 m; ρ_p = 8500 kg/m^3; M = 1.92; $\Delta\gamma_x$ = 1.8°; $\Delta\gamma_\text{w}$ = 4.0°; μ_0 = 0.3; μ_d = 0.2) and [24] ($\text{Re}_\text{D} = 10^5$; D_i = 30 μm; r_0 = 0.0168 m; ρ_p = 3960 kg/m^3; M = 0.34; $\Delta\gamma_x$ = 0.9°; $\Delta\gamma_\text{w}$ = 4.0°; μ_0 = 0.5; μ_d = 0.5). In the first case we have a substantially nonequilibrium mode of flow of a gas with suspended particles characterized by a strong shock interaction between the particles and the tube walls. The second refers to the mode of a locally nonequilibrium flow for which small velocities of slipping of the phases and a low level of the rate of collision between the particles and the wall are typical. Figure 3 shows distributions of the velocity of the gas w_x and the velocity of the disperse phase $w_{\text{p}x}$ calculated by the relation

$$w_{\text{p}x} = \left. \frac{\sum\limits_{i=1}^{N} (\tilde{w}_{xi}, \dot{m}_i)}{\sum\limits_{i=1}^{N} \dot{m}_i} \right|_{x=\text{const}} \tag{21}$$

TABLE 3. Coefficients of Friction, Pressure Gradients, and Intensities of Collision of Particles with the Tube Wall

Models	[12]	[13]	[14]	[15]
Experimental conditions of [23] (M = 1.92)				
c_f	0.00611	0.00551	0.00532	0.00607
$\frac{\partial P}{\partial x}$	1070	892	878	1011
J_w	0.78	0.48	0.52	0.62
Experimental conditions of [24] (M = 0.34)				
c_f	0.00471	0.00427	0.00420	0.00471
$\frac{\partial P}{\partial x}$	741	615	605	671
J_w	0.07	0.045	0.05	0.06

for a stabilized portion of flow in a tube, when different models of turbulence are used, and test data of [23, 24]. It is seen that use of the different versions of the k–ε model gives acceptable results in predicting the velocity of the carrying medium. The models of [15, 12], which provide a higher level of turbulence energy when $r > 0.5r_0$ compared to the modifications of [14, 13], give values for the disperse phase that are closer to the experiment (Fig. 4). An increase in the energy of gas turbulence facilitates enhancement of the transverse motion of the particles, which in this case leads to leveling of the values of the averaged velocity of the disperse phase along the tube radius.

Values of the coefficient of friction c_f, the pressure gradient $(-\partial P/\partial x)$, and the rate of collision between the particles and the tube wall $J_w = \frac{1}{N_T S}\sum_{i=1}^{N} \dot{m}_i$ (where S is the surface area of the tube wall bounded by two successive cross sections along x where the number of collisions N is calculated) are presented in Table 3 for the considered modes of turbulent flow of a gas suspension in a tube. As follows from the table, the modifications of [14, 13], which predict a lower level of turbulent pulsations in the near-wall region, provide smaller values of the coefficient of friction of the gas and the rate of collision of the particles with the wall and consequently a lower value of the pressure gradient compared to the cases where the models of [15, 12] were used.

The results obtained indicate the necessity of careful modeling of the intensity of turbulent pulsations of the gas even for substantially nonequilibrium flows of the gas suspension, when $\tau_i >> \tau_{lag}$. It is also necessary to note the closeness of results obtained based on models of different levels of closing ([12] and [15]), which indicates a weak effect of the disperse phase on the scale L. Moreover, a special study of the effect of anisotropy of the turbulent pulsations of the gas on the particle flow was carried out. In this case, the values of the normal Reynolds stresses $\overline{\rho w_x'^2}$, $\overline{\rho w_r'^2}$, and $\overline{\rho w_\varphi'^2}$ were determined from the solution of the corresponding transport equations [25] modified for two-phase flows. However, the data obtained did not show a noticeable contribution of this method of determining the turbulent pulsations of the gas to the calculated distributions of the dynamic characteristics of the ensemble of particles.

Thus, a mathematical model of the turbulent flow of a gas suspension in a tube has been constructed within the framework of the combined Euler–Lagrange method of description of a mixture. The model was tested on experimental data for different modes of flow of a two-phase medium and showed good agreement with them. Moreover, the results of the calculations are in satisfactory correlation with theoretical data obtained using the continuous model [1].

It is found on the basis of a comparative analysis of calculated and experimental data that due a weak effect of the disperse phase on the turbulence scale, in modeling the turbulent structure of the carrying medium it suffices to use a one-parameter model of turbulence that contains only one differential equation for the kinetic energy of turbulent fluctuations k that includes additional dissipation due to the presence of the particles in the flow.

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NOTATION

w_x and w_r, components of the averaged velocity of the gas; x, r, φ, cylindrical coordinates; ρ and P, density and pressure; μ_g and μ_T, molecular and turbulent viscosity of the gas; U_0, mean velocity in the tube cross section; g, acceleration of gravity; F_x, force of interphase interaction; $\vec{\tilde{z}}_i$, vector of the position of the representative of the i-th group of particles ($i = 1, ..., N$); $\vec{\tilde{w}}_i$ and $\vec{\tilde{\omega}}_i$, vectors of the linear and angular velocity of the representative of the i-th group of particles; m_p and I_p, mass and moment of inertia of the particle; t, time; $\vec{F}_{di}$, force of aerodynamic resistance; $\vec{F}_{Ii}$, inertial component of the force of interphase interaction between the gas and the i-th particle; $\vec{F}_{Mi}$, Magnus force; $m_p\vec{g}$, force of gravity; $\vec{M}_i$, torque; D_i, particle diameter; ρ_p, density of the particle material; $\vec{w} = \vec{\bar{w}} + \vec{w}'$, actual values of the components of the gas velocity; L and k, scale and kinetic energy of turbulence; $\varepsilon = \bar{\varepsilon} + \hat{\varepsilon}$, total dissipation of turbulence energy; $\bar{\varepsilon}$, isotropic component of ε; $\rho\varepsilon_{add}$ and $\rho\Phi_{add}$, terms allowing for the effect of the disperse phase on the turbulent parameters of the carrying medium; Π_k and Π_ε, additional redistributing terms in the equations for the turbulence energy k and its dissipation ε; N_T, number of realizations of the calculation of the motion of the representative particle of the i-th group; r_0, tube radius; $\vec{w}'$, pulsation component of the gas velocity; Ω_φ, projection of the vector of gas vorticity onto the circumferential direction φ; τ_T, time interval; τ_{lag}, lifetime of the turbulent vortex; τ_{rel}, time of residence of the particle in the vortex; N, number of representative particles passing through the volume $V^{(k,j)}$ in the time Δt_i; $\dot{m}_i$, mass flow rate of the i-th group of particles; τ_i, time of relaxation of the i-th particle; Re_D, dimensionless Reynolds number; M, parameter of loading of the flow with the disperse phase ($M = G_p/G_g$, ratio of the flow rates of the phases); $\Delta\gamma_w$, mean value of the angle of surface roughness in the circumferential direction φ; w_x, gas velocity; w_{px}, velocity of the disperse phase; w_{x0}, gas velocity on the tube axis; $(-\partial P/\partial x)$, pressure gradient; J_w, rate of collision of the particles with the tube wall. Subscripts: g, gas; T, turbulent; p, particle; d, dynamic; lag, Lagrangian; rel, relative; D, diametral; 1 and 2, parameters of the particle before and after collision with the wall surface, respectively.

REFERENCES

1. A. V. Starchenko, A. M. Bubenchikov, and E. S. Burlutskii, *Teplofizika Aéromekhanika*, No. 1, 59-71 (1999).
2. A. A. Shraiber, L. B. Gavin, V. A. Naumov, and V. P. Yatsenko, *Turbulent Flows of Gas Suspensions* [in Russian], Kiev (1987).
3. Z. R. Gorbis, *Heat Transfer and Hydromechanics of Disperse Through Flows* [in Russian], Moscow (1970).
4. R. I. Nigmatulin, *Dynamics of Multiphase Media* [in Russian], Pt. 1, Moscow (1987).
5. L. V. Kondrat'ev, *Modelirov. Mekhanike* (Novosibirsk), **19**, No. 6, 55-61, (1988).
6. V. A. Naumov, A. M. Podvysotsky, and A. A. Shraiber, in: *Proc. 1st Int. Symp. Two-Phase Flow Modeling and Experimentation*, Rome, Italy, October 9-11, 1995, Vol. 1, pp. 109-116.
7. L. I. Zaichik, *Inzh.-Fiz. Zh.*, **63**, No. 4, 404-413 (1992).
8. I. V. Derevich, in: *Proc. 3rd Int. Minsk Forum "Heat and Mass Transfer–MIF-96"* [in Russian], Minsk, May 20-24, 1996, Vol. 5, Minsk (1996), pp. 134-141.
9. C. T. Crowe, M. P. Sharma, and D. E. Stock, *Trans. ASME, D* [Russian translation], **99**, No. 2, 150-159 (1977).
10. A. A. Mostafa, H. C. Mongia, V. G. McDonell, and G. S. Samuelsen, *AIAA J.*, No. 2, 167-183 (1989).
11. N. Huber and M. Sommerfeld, *Powder Technology*, **99**, 90-101 (1998).
12. O. F. Vasil'ev and V. I. Kvon, *Zh. Prikl. Mekh. Tekh. Fiz.*, No. 6, 132-140 (1971).
13. B. E. Launder and B. I. Sharma, *Lett. Heat Mass Transfer*, No. 2, 131-138 (1974).

14. K. Y. Chien, *AIAA J.*, **20**, No. 1, 33-38 (1982).
15. C. B. Hwang and C. A. Lin, *AIAA J.*, **36**, No. 1, 38-43 (1998).
16. S. Patankar, *Numerical Heat Transfer in Fluid Flow*, Hemisphere Publishing Corp., New York (1980).
17. S. Matsumoto and S. Saito, *J. Chem. Eng Japan*, **3**, 223-230 (1970).
18. N. N. Koshkin and M. G. Shirkevich, *Handbook on Elementary Physics* [in Russian], Moscow (1988).
19. V. A. Lashkov, *Inzh.-Fiz. Zh.*, **60**, No. 2, 197-203 (1991).
20. L. M. Simuni, *Inzh.-Fiz. Zh.*, **10**, No. 1, 86-91 (1966).
21. G. E. Forsythe, M. A. Malcolm, and C. B. Moler, *Computer Methods for Mathematical Computations*, Prentice-Hall Inc., Englewood Cliffs, New York (1977).
22. V. I. Bukreev and V. M. Shakhin, *Statistically Nonstationary Turbulent Flow in a Tube* [in Russian], Novosibirsk (1981), Dep. at VINITI 16.02.81, No. 866-881.
23. A. S. Mul'gi, in: *Turbulent Two-Phase Flows* [in Russian], Tallinn (1979), pp. 47-59.
24. M. K. Laats and A. S. Mul'gi, in: *Turbulent Two-Phase Flows* [in Russian], Tallinn (1979), pp. 32-46.
25. M. Prud'homme and S. Elghobashi, *Num. Heat Transfer*, **10**, No. 4, 349-368 (1986).